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## Electrostatic Potential and Capacitance

### Objectives

After going through this lesson, the learners will be able to

- Understand the quantitative effect of the dimensions of the capacitor on its capacity
- Visualize the role of dielectric filling the gap between the plates a capacitor by a dielectric on the charge stored
- Know a relation between capacitance and potential difference
- Conceptualize Electric field between the plates

### Content Outline

- Unit Syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Capacitance of a parallel plate capacitor without dielectric in between the plates
- Capacitance of a parallel plate capacitor with a dielectric in between the plates
- Capacitance of a parallel plate capacitor partially filled with a dielectric slab
- Energy stored in a Capacitor
- Loss of energy during redistribution of charges
- Summary

### Unit Syllabus

#### Chapter-1: Electric Charges and Fields

Electric Charges; Conservation of charge, Coulomb's law-force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

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## Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

### Module Wise distribution of Unit Syllabus - 11 Modules

The above unit is divided into 11 modules for better understanding.

Module 1	<ul style="list-style-type: none"><li>● Electric charge</li><li>● Properties of charge</li><li>● Coulombs' law</li><li>● Characteristics of coulomb force</li><li>● Constant and the intervening medium</li><li>● numerical</li></ul>
Module 2	<ul style="list-style-type: none"><li>● Forces between multiple charges</li><li>● Superposition</li><li>● Continuous distribution of charges</li><li>● numerical</li></ul>
Module 3	<ul style="list-style-type: none"><li>● Electric field E</li><li>● Importance of field and ways of describing field</li><li>● Point charges superposition of electric field</li><li>● numerical</li></ul>
Module 4	<ul style="list-style-type: none"><li>● Electric dipole</li><li>● Electric field of a dipole</li><li>● Charges in external field</li><li>● Dipole in external field Uniform and non-uniform</li></ul>
Module 5	<ul style="list-style-type: none"><li>● Electric flux,</li><li>● Flux density</li><li>● Gauss theorem</li><li>● Application of gauss theorem to find electric field</li></ul>

	<ul style="list-style-type: none"> <li>● For a distribution of charges</li> <li>● Numerical</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>● Application of gauss theorem Field due to field infinitely long straight wire</li> <li>● Uniformly charged infinite plane</li> <li>● Uniformly charged thin spherical shell (field inside and outside)</li> <li>● Graphs</li> </ul>
Module 7	<ul style="list-style-type: none"> <li>● Electric potential,</li> <li>● Potential difference,</li> <li>● Electric potential due to a point charge, a dipole and system of charges;</li> <li>● Equipotential surfaces,</li> <li>● Electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.</li> <li>● Numerical</li> </ul>
Module 8	<ul style="list-style-type: none"> <li>● Conductors and insulators,</li> <li>● Free charges and bound charges inside a conductor.</li> <li>● Dielectrics and electric polarization</li> </ul>
Module 9	<ul style="list-style-type: none"> <li>● Capacitors and capacitance,</li> <li>● Combination of capacitors in series and in parallel</li> <li>● Redistribution of charges , common potential</li> <li>● numerical</li> </ul>
Module 10	<ul style="list-style-type: none"> <li>● Capacitance of a parallel plate capacitor with and without dielectric medium between the plates</li> <li>● Energy stored in a capacitor</li> </ul>
Module 11	<ul style="list-style-type: none"> <li>● Typical problems on capacitors</li> </ul>

## Module 10

### Words You Must Know

Let us recollect the words we have been using in our study of this physics course.

- **Electric Charge:** Electric charge is an intrinsic characteristic of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions.

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- **Conductors:** Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called conductors. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are all conductors of electricity.
  - **Insulators:** Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called insulators.
  - **Point Charge:** When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as point charges.
  - **Coulomb's Law of electrostatics:** The force of attraction, or repulsion, between two point charges, is directly proportional to the product of the magnitude of the two charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.
  - **Coulomb's Force:** It is the electrostatic force of interaction between the two point charges. Or distribution of charges
  - **Electrostatic force:** Like charges repel each other and unlike charges attract each other, this force of attraction, or repulsion, between two charges, is called electrostatic force ,this is also Coulomb's force.
  - **Linear charge density:** The linear charge density  $\lambda$  is defined as the charge per unit length.
  - **Surface charge density:** The surface charge density  $\sigma$  is defined as the charge per unit surface area.
  - **Volume charge density:** The volume charge density  $\rho$  is defined as the charge per unit volume.
  - **Superposition Principle:** For an assembly of charges  $q_1, q_2, q_3, \dots$ , the force on any charge, say  $q_1$ , is the vector sum of the force on  $q_1$  due to  $q_2$ , the force on  $q_1$  due to  $q_3$ , and so on. For each pair, the force is given by Coulomb's law for two point charges.
  - **Electric Field:** A region around a charged particle or object within which a force would be experienced by a charged particle or object.
  - **Source and test charge:** The charge, which is producing the electric field, is called a source charge and the charge, which tests the effect of a source charge, is called a test charge.

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- **Uniform Field:** A uniform electric field is one whose magnitude and direction is same at all points in space and it will exert same force of a charge regardless of the position of charge.
  - **Non uniform field:** we know that electric field of point charge depends upon location of the charge. Hence has different magnitude and direction at different points. We refer to this field as non-uniform electric field
  - **Principle of superposition of fields:** Electric field intensity  $E$  at any point  $P$  due to all  $n$  point charges will be equal to the vector sum of electric field intensities  $E_1, E_2, E_3, \dots, E_n$  produced by individual charges at the point  $P$ . Hence  $E = E_1 + E_2 + \dots + E_n$
  - **Torque:** Torque is the tendency of a force to rotate an object about an axis.
  - **Electric field lines:** An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of the electric field at that point.
  - **Surface charge density in terms of area element:** The surface charge density  $\sigma$  at the area element  $\Delta s$  is given by  $\sigma = \frac{\Delta Q}{\Delta s}$
  - **Area vector:** The area element vector  $\Delta S$  at a point on a closed surface equals  $\Delta S \hat{n}$  where  $\Delta S$  is the magnitude of the area element and  $\hat{n}$  is a unit vector in the direction of outward normal at that point.
  - **Gauss's law:** The flux of the electric field through any closed surface  $S$  is  $1/\epsilon_0$  times the total charge enclosed by that surface.
  - **Gaussian surface:** The closed surface that we need to choose for applying Gauss's law to a particular charge distribution is called the Gaussian surface.
  - **Capacitance:** It measures the ability to store charge and it is equal to the amount of charge required to have a potential difference of 1V between the plates of the capacitor.
  - **Electrostatic potential:** It is the amount of work done in moving a unit positive charge, from infinity to that point, in the electric field, without accelerating it.
  - **Series grouping of capacitors:**
    - In series combination,
      - The charge on each capacitor is the same.
      - The reciprocal of equivalent capacitance, of a series combination, is equal to the sum of, the reciprocals, of the individual capacitances.
  - **Parallel grouping of capacitors:**
    - In parallel grouping
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- The equivalent capacitance is equal to the sum of the individual capacitances.
- The potential difference across each capacitor is the same.
- The charge on each capacitor is proportional to its capacitance.
- **Common potential:** When conductors charged to different potential are connected then charge will flow from a body at higher potential to the one at lower potential till both reach at the same potential called common potential.

### Introduction

The capacitance of a conductor depends upon the shape and size of the conductor and the surrounding medium. Let us find out the factor on which the capacitance of a parallel plate capacitor depends.

A parallel plate capacitor consists of two large planes parallel conducting plates separated by a small distance.

Let us first take the medium between the plates to be vacuum.

### Capacitance of a Parallel Plate Capacitor without Dielectric in between the Plates

Let  $A$  be the area of each plate and  $d$  the separation between the plates. Plate 1 has a uniform surface charge density  $\sigma = \frac{Q}{A}$  and the plate 2 has a uniform surface charge density  $-\sigma$ . As  $d^2 \ll A$  we can use the result, for the electric field due to an infinite plane sheet, of uniform surface charge density.

i.e.  $E = \frac{\sigma}{2\epsilon_0}$

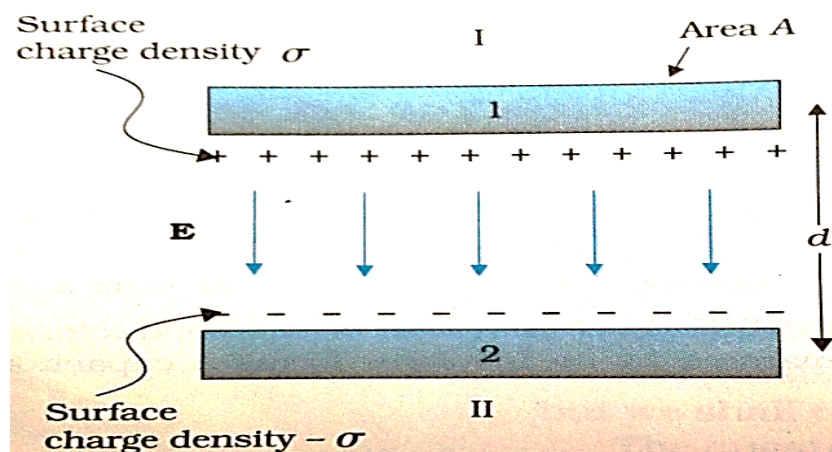


Figure: The parallel plate capacitor

In the outer region I

$$E_{\text{net}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0 \quad (1)$$

As the electric field due to two plates is in opposite directions

In the other region II also

$$E_{\text{net}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0 \quad (2)$$

In the inner region between the plates, the electric field due to both the plates is from the positive to the negative plate, and it is uniform throughout as  $d^2 \ll A$ . Its magnitude is

$$E_{\text{net}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \sigma/\epsilon_0$$

$$\text{As } V = E d = \frac{Qd}{A\epsilon_0}$$

So  $C = Q/V = \frac{A\epsilon_0}{d}$

**Thus the capacitance of a parallel plate capacitor depends**

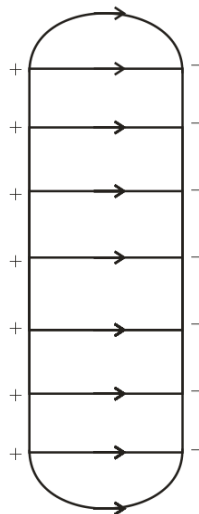
- **Area of the plates ( $C \propto A$ )**
- **Distance between the plates ( $C \propto \frac{1}{d}$ )**
- **Permittivity of the medium between the plates ( $C \propto \epsilon$ )**

**[For video on capacitance of a parallel plate capacitor refers to the video:**

<https://www.khanacademy.org/science/physics/circuits-topic/circuits-with-capacitors/v/capacitance>]

**Note:-**

For plates of finite area, the field between the plates will not be (strictly speaking) uniform throughout. The field line bend outward at the edges, - an effect called “fringing of the field”



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**The unit of capacitance, Farad, is a very big unit.**

In order to see how large one farad is , let us calculate the area of the plates needed to have  $C = 1 F$ , for  $d = 1 \text{ cm}$ .

We have 
$$A = \frac{Cd}{\epsilon_0}$$
$$= \frac{1 \times 10^{-2}}{8.85 \times 10^{-12}} m^2 = 10^9 m^2$$

This is **the area of a square plate of side (nearly) equal to 30 Km**

### **Capacitance of a Parallel Plate Capacitor with Dielectric in between the Plates**

Let us see how the capacitance of a parallel plate capacitor is modified when the space between

Its plates are filled with dielectric.

The two plates are large, each of area  $A$ , separated by a distance  $d$ , and the charges on the two plates are  $+Q$  and  $-Q$ , respectively. When there is vacuum between the plates, we have

$$E_0 = \frac{\sigma}{\epsilon_0}$$

And the potential difference  $V_0$  is

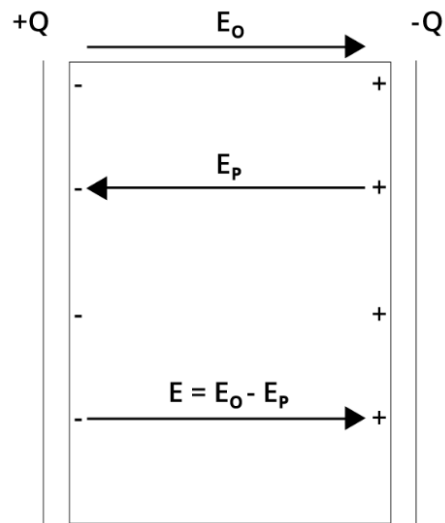
$$V_0 = E_0 d.$$

The capacitance  $C_0$  in this case is 
$$C_0 = \frac{Q}{V_0} = \frac{A\epsilon_0}{d} \quad (1)$$

Consider a dielectric, inserted between the plates, fully occupying the intervening region.

The dielectric as you may recall is an insulator .The dielectric gets polarized by the field and a positive charge appears on the face of the dielectric facing the negative plate of the capacitor and vice versa.





The induced electric field due to the polarization of the dielectric is opposite to the original electric field between the plates of the capacitor. The net electric field between the plates.

$$\vec{E} = \vec{E}_0 + \vec{E}_p$$

$$|\vec{E}| = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$$E = \frac{\sigma - \sigma_p}{\epsilon_0}$$

As  $E_0/E = K$ , where  $K$  is the dielectric constant of the medium.

$$\therefore E = \frac{E_0}{K} = \frac{\sigma}{\epsilon_0 K}$$

Thus the potential difference between the plates will now be

$$\begin{aligned} V = Ed &= \frac{\sigma d}{\epsilon_0 K} \\ &= \frac{Qd}{A\epsilon_0 K} \end{aligned}$$

The capacitance  $C$ , with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{A\epsilon_0 K}{d} \quad (2)$$

$$C = \mathbf{K}C_0$$

The product  $\epsilon_0 k$  is called the **permittivity of the medium** and is denoted by  $\epsilon$ .

$$\epsilon = \epsilon_0 K$$

$$K = \frac{\epsilon}{\epsilon_0}$$

**K is called the dielectric constant of the substance and is a dimensionless ratio.**

From eq. 1 & 2 we see that

$$C/C_0 = K \quad (3)$$

**So Dielectric constant can, therefore, also be defined as the ratio of the capacitance of a parallel plate capacitor, with the given medium as the medium of separation between the plates, to the capacitance of the same capacitor, with vacuum as the medium of separation between the plates.**

As  $K > 1$ ,

The capacitance of a capacitor **increases** from its vacuum value when the space between its plates is filled with a dielectric.

For video on the effect of dielectric on the capacitance of a capacitor refer to the video: <https://www.khanacademy.org/science/physics/circuits-topic/circuits-with-capacitors/v/dielectrics-capacitors>

[https://www.youtube.com/watch?v=rkntp3\\_cZl4](https://www.youtube.com/watch?v=rkntp3_cZl4)

### Capacitance of a Parallel Plate Capacitor partially filled with a Dielectric Slab

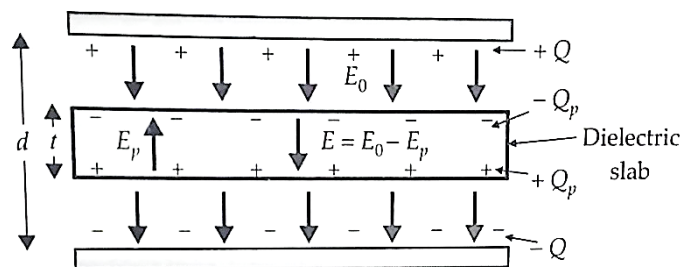
The capacitance of a parallel plate capacitor, of plate area A and plate separation d with vacuum between its plates is given by

$$C_0 = \frac{\epsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are  $\pm Q$ . Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

When a dielectric slab of thickness t ( $t < d$ ), is placed between the plates, the field  $E_0$  polarizes the dielectric. This induces a charge  $-Q_p$  on the upper surface and  $+Q_p$  on the lower



surface of the dielectric. These induced charges set up a field,  $E_p$ , inside the dielectric in the opposite direction of  $\vec{E}_0$ . The induced field is given by

$$E_p = \frac{\sigma_p}{\epsilon_0}$$

The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{K}$$

Where K is the dielectric constant of the slab.

So, between the capacitor plates, the field E exists over a distance t and field  $E_0$  exists over the remaining distance (d-t).

Hence the potential difference between the capacitor plates is

$$\begin{aligned} V &= E_0(d - t) + Et = E_0(d - t) + \frac{E_0}{K}t && \left[ \because \frac{E_0}{E} = K \right] \\ &= E_0 \left( d - t + \frac{t}{K} \right) = \frac{Q}{\epsilon_0 A} \left( d - t + \frac{t}{K} \right) \end{aligned}$$

The capacitance of the capacitor on partial introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

### Special Case

If the dielectric fills the entire space between the plates, then  $t = d$ , and we get

$$C = \frac{\epsilon_0 A}{d}. \quad K = KC_0$$

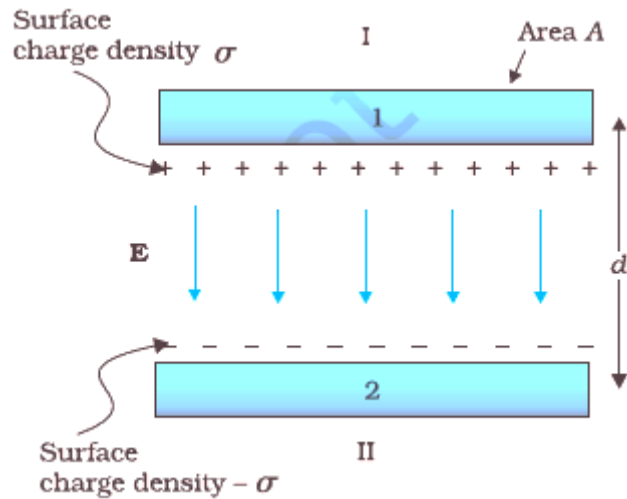
Thus the capacitance of the parallel plate capacitor increases  $\kappa$  times when its entire space is filled with a dielectric material.

Clearly,  $K = \frac{C}{C_0}$

### Energy Stored in a Capacitor

A capacitor is a device to store energy. The process of charging a capacitor involves the transferring of charges from its one plate to another. The work done in charging the capacitor is stored in the form of electrical potential energy. This energy is supplied by the battery at the expense of its chemical energy. This energy can be recovered by allowing the capacitor to discharge.

### Expression for the energy stored in a Capacitor



### The parallel plate capacitor

Consider a capacitor of capacitance  $C$ . Initially, its two plates are uncharged. Suppose the positive charge is transferred, from plate 2 to plate 1, bit by bit. In this process, external work has to be done because; at any stage plate 1 is at higher potential than plate 2.

Suppose at any instant, the plates 1 and 2 have charges  $+q$  and  $-q$  respectively, as shown in the figure. Then the potential difference between the two plates, at that instant, will be

$$v = \frac{q}{C}$$

#### Consider

- a. **Work done in transferring charge  $dq$  from plate 2 to plate 1.**
- b. **Total work done in charging the capacitor may be considered as the energy stored in the electric field between the plates.**

Suppose now a small additional charge  $dq$  be transferred from plate 2 to plate 1. The work done will be

$$dW = v dq = \frac{q}{C} \cdot dq$$

The total work done, in transferring a charge,  $Q$ , from plate 2 to plate 1, [Figure (b)] will be

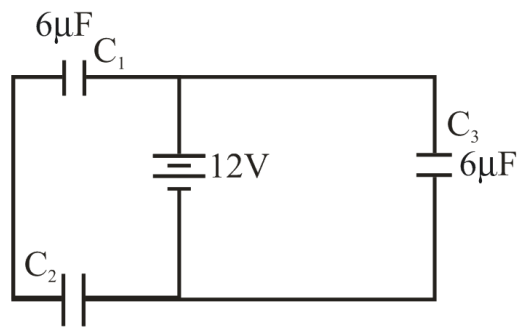
$$W = \int_0^Q \frac{q}{C} \cdot dq = \left[ \frac{q^2}{2C} \right]_0^Q = \frac{1}{2} \cdot \frac{Q^2}{C}$$

This work done is stored as electrical potential energy,  $U$ , of the capacitor.

$$U = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} \cdot CV^2 = \frac{1}{2} QV \quad [ \because Q = CV ]$$

#### Example:

For the 'set up', shown in the figure,



**Calculate**

- i. equivalent capacitance of the combination
- ii. the charge on each capacitor
- iii. Energy stored in the network
- iv. Energy stored in individual capacitor

Solution:

$C_1$  and  $C_2$  are connected in series

$$\begin{aligned} \text{(i)} \quad \frac{1}{c_{12}} &= \frac{1}{c_1} + \frac{1}{c_2} \\ &= \frac{1}{6} + \frac{1}{6} = 2/6 \\ &= \frac{1}{3} \end{aligned}$$

$$C_{12} = 3\mu\text{F}$$

$$C = c_{123} = C_{12} + C_3 = (3 + 6)\mu\text{F} = 9\mu\text{F}$$

(ii) Charge on each

$$q_1 = q_2 = C_{12}V = 3 \times 10^{-6} \times 12\text{C} = 36 \times 10^{-6}\text{C} = 36\mu\text{C}$$

$$q_3 = C_3V = 6 \times 12\mu\text{C} = 72\mu\text{C}$$

(iii) Total Energy stored in the network

$$= \frac{1}{2}CV^2 = (1/2) \times 9 \times 10^{-6} \times 12 \times 12$$

$$= 648 \times 10^{-6}$$

(iv) Energy stored in  $C_1$

$$= (1/2) q_1^2 / C_1 = \frac{1}{2} \times \frac{(36 \times 10^{-6})^2}{6 \times 10^{-6}}$$

$$= 108 \times 10^{-6}\text{J}$$

$$= 108 \mu J$$

$$\text{energy stored in } C_2 = \frac{1}{2} \frac{q_2^2}{c_2} = \frac{1}{2} \frac{(36 \times 10^{-6})^2}{2 \times 6 \times 10^{-6}}$$

$$= 108 \mu J$$

energy stored in  $C_3$

$$= \frac{1}{2} \times 6 \times 10^{-6} \times 12^2 J = 432 \mu J$$

### Effect of dielectric when the battery is kept disconnected from the capacitor.

A parallel plate capacitor has a capacitance  $C_0$ , charge  $Q_0$  potential difference  $V_0$ , Electric field  $E_0$  and energy stored  $U_0$  before the dielectric slab is inserted. Then

$$Q_0 = C_0 V_0, E_0 = \frac{V_0}{d}, U_0 = \frac{1}{2} C_0 V_0^2$$

**The battery is disconnected and a dielectric slab of thickness equal to the distance between the plates is introduced between the plates then:**

- (i) Charge: The charge on the capacitor plates remains  $Q_0$  because the battery has been disconnected before the insertion of the dielectric slab.
- (ii) Electric field: When the dielectric slab is inserted between the plates, the induced surface charge on the dielectric reduces the field to a new value given by

$$E = \frac{E_0}{K}$$

- (iii) Potential difference. The reduction in the electric fields results in the decrease in potential difference.
- (iv) Capacitance as a result of the decrease in potential difference, the capacitance increases  $\kappa$  times.

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/K} = K \frac{Q_0}{V_0} = K C_0$$

- (v) Energy stored. The energy stored decreases by a factor of  $\kappa$ .

$$U = \frac{1}{2} C V^2 = \frac{1}{2} (K C_0) \left( \frac{V_0}{K} \right)^2 = \frac{1}{K} \cdot \frac{1}{2} C_0 V_0^2 = \frac{U_0}{K}.$$

### Effect of dielectric when battery remains connected across the Capacitor:-

A parallel plate capacitor has a capacitance  $C_0$ , charge  $Q_0$  potential difference  $V_0$ , Electric field  $E_0$  and energy stored  $U_0$  before the dielectric slab is inserted. Then

$$Q_0 = C_0 V_0, E_0 = \frac{V_0}{d}, U_0 = \frac{1}{2} C_0 V_0^2$$

**The battery is still connected and a dielectric slab of thickness equal to the distance between the plates is introduced between the plates then:**

(i) Potential difference. As the battery remains connected across the capacitor, the potential difference remains constant at  $V_0$  even after the introduction of dielectric slab.

(ii) Capacitance. The capacitance increases from  $C_0$  to  $C$ .

$$C = K C_0$$

(iii) Charge. The charge on the capacitor plates increases from  $Q_0$  to  $Q$ .

$$Q = CV = K C_0 V_0 = K U_0.$$

(iv) Electric field. As the potential difference remains unchanged, so the electric field  $E_0$  between the capacitor plates remains unchanged.

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0$$

(v) Energy stored. The energy stored in the capacitor increases  $\kappa$  times.

$$U = \frac{1}{2} CV^2 = \frac{1}{2}(K)V_0^2 = K \frac{1}{2} C_0 V_0^2 = K U_0.$$

**Effect of dielectric on various parameters when the entire space between the plates is filled with a dielectric of dielectric constant  $K$**

Battery disconnected from the capacitor	Battery kept connected across the capacitor
$Q = Q_0$ (constant)	$Q = K Q_0$
$V = \frac{V_0}{k}$	$V = V_0$ (constant)
$E = \frac{E_0}{K}$	$E = E_0$ (constant)
$C = K C_0$	$C = K C_0$
$U = \frac{U_0}{K}$	$U = K U_0$

### Loss of Energy during Redistribution of Charges

Let  $C_1$  and  $C_2$  be the capacitances and  $V_1$  and  $V_2$  be the potentials of the two conductors before they are connected together. Potential energy before connection is

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

After connection, let  $V$  be their common potential. Then

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Potential energy after connection is

$$U_f = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2} (C_1 + C_2) V^2$$

$$\begin{aligned} & \frac{1}{2} (C_1 + C_2) \left[ \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right]^2 \\ &= \frac{1}{2} \cdot \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)} \end{aligned}$$

Loss in Energy,  $U = U_i - U_f$

$$\begin{aligned} &= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2} \cdot \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)} \\ &= \frac{1}{2(C_1 + C_2)} + C_2^2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2 - 2 C_1 C_2 V_1 V_2 \\ &= \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} [V_1^2 + V_2^2 - 2 V_1 V_2] \\ &= \frac{1}{2} \cdot \frac{C_1 C_2 (V_1 - V_2)^2}{C_1 + C_2} \end{aligned}$$

This is always positive whether  $V_1 > V_2$  or  $V_1 < V_2$ .

So

**When two charged conductors are connected, charges flow from higher potential to lower potential till the potentials of the two conductors become equal. In doing so, there is always some loss of potential energy in the form of heat and electromagnetic radiation due to the flow of charges in connecting wires.**

### Energy stored in a series combination of Capacitors

For a series combination,  $Q = \text{constant}$

Total energy,

$$U = \frac{Q^2}{2} \cdot \frac{1}{C} = \frac{Q^2}{2} \cdot \left[ \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \right]$$

$$U = U_1 + U_2 + U_3 + \dots$$

**Total Energy stored in a series combination of capacitors is the sum of energy stored in the individual capacitors.**

Energy stored in a parallel combination of capacitors

For a parallel combination,  $V = \text{constant}$



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### Total Energy

$$U = \frac{1}{2}CV^2 = \frac{1}{2}[C_1 + C_2 + C_3 + \dots]V^2$$
$$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + \dots$$

Or  $U = U_1 + U_2 + U_3 + \dots$

**The energy stored, in a parallel grouping of capacitors, is the sum of energy stored in the individual capacitors.**

Hence

**Total energy is additive both for the series and the parallel combinations of capacitors.**

### Summary

- **Capacitance of a parallel plate capacitor depends:**
  - Area of the plates ( $C \propto A$ )
  - Distance between the plates ( $c \propto \frac{1}{d}$ )
  - Permittivity of the medium between the plates ( $c \propto \epsilon$ )
- **The capacitance C, with dielectric between the plates, is**

$$C = \frac{Q}{V} = \frac{A\epsilon_0 K}{d}$$

Capacitance of a capacitor increases from its vacuum value when the space is filled with a dielectric.  $C = K C_0$

- **The energy stored in a capacitor**  $U = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} \cdot CV^2 = \frac{1}{2}QV$
- The energy stored in a parallel grouping of capacitors is the sum of energy stored in the individual capacitor.
- The energy stored in a parallel grouping of capacitors is the sum of energy stored in the individual capacitor.
- The energy stored in a parallel grouping of capacitors is the sum of energy forced in the individual capacitor.
- Loss of energy when two capacitors charged to different potentials are connected in parallel is  $U = \frac{1}{2} \cdot \frac{C_1 C_2 (V_1 - V_2)^2}{C_1 + C_2}$
- Effect of dielectric on various parameters when a dielectric slab of dielectric constant  $k$  fills the entire space between the plates

Battery disconnected from the capacitor	Battery kept connected across the capacitor
$Q=Q_0$ (constant)	$Q = K Q_0$
$V = \frac{V_0}{K}$	$V = V_0$ (constant)
$E = \frac{E_0}{K}$	$E = E_0$ (constant)
$C = K C_0$	$C = K C_0$
$U = \frac{U_0}{K}$	$U = K U_0$